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Cambridge International AS & A Level

MATHEMATICS (9709) P3

TOPIC WISE QUESTIONS + ANSWERS | COMPLETE SYLLABUS

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Chapter 8

Differential equations

282. 9709_s20_qp_31 Q: 8

A certain curve is such that its gradient at a point (x, y) is proportional to $\frac{y}{x\sqrt{x}}$. The curve passes through the points with coordinates $(1, 1)$ and $(4, e)$.

(a) By setting up and solving a differential equation, find the equation of the curve, expressing y in terms of x. [8]

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(b) Describe what happens to y as x tends to infinity. [1]

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283. 9709_s20_qp_32 Q: 7

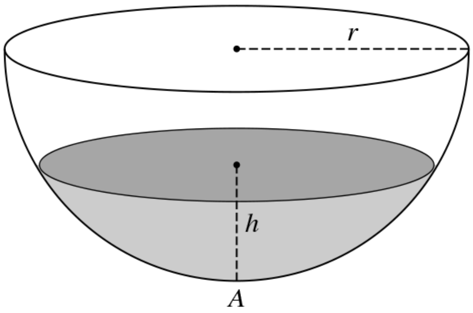
The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = \frac{y-1}{(x+1)(x+3)}.$$

It is given that $y = 2$ when $x = 0$.

Solve the differential equation, obtaining an expression for y in terms of x. [9]

284. 9709_s20_qp_33 Q: 10



A tank containing water is in the form of a hemisphere. The axis is vertical, the lowest point is A and the radius is r, as shown in the diagram. The depth of water at time t is h. At time $t = 0$ the tank is full and the depth of the water is r. At this instant a tap at A is opened and water begins to flow out at a rate proportional to $\sqrt{h}$. The tank becomes empty at time $t = 14$.

The volume of water in the tank is V when the depth is h. It is given that $V = \frac{1}{3}\pi(3rh^2 - h^3)$.

(a) Show that h and t satisfy a differential equation of the form

$$\frac{dh}{dt} = -\frac{B}{2rh^{\frac{1}{2}} - h^{\frac{3}{2}}},$$

where B is a positive constant. [4]

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(b) Solve the differential equation and obtain an expression for t in terms of h and r. [8]

285. 9709_w20_qp_31 Q: 8

The coordinates (x, y) of a general point of a curve satisfy the differential equation

$$x\frac{dy}{dx} = (1 - 2x^2)y,$$

for $x > 0$. It is given that $y = 1$ when $x = 1$.

Solve the differential equation, obtaining an expression for y in terms of x. [6]

286. 9709_w20_qp_32 Q: 7

The variables x and t satisfy the differential equation

$$e^{3t}\frac{dx}{dt} = \cos^2 2x,$$

for $t \geqslant 0$. It is given that $x = 0$ when $t = 0$.

(a) Solve the differential equation and obtain an expression for x in terms of t. [7]

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(b) State what happens to the value of x when t tends to infinity. [1]

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287. 9709_m19_qp_32 Q: 6

The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = ky^3 e^{-x},$$

where k is a constant. It is given that $y = 1$ when $x = 0$, and that $y = \sqrt{e}$ when $x = 1$. Solve the differential equation, obtaining an expression for y in terms of x. [7]

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288. 9709_s19_qp_31 Q: 5

(i) Differentiate $\dfrac{1}{\sin^2\theta}$ with respect to θ. [2]

(ii) The variables x and θ satisfy the differential equation

$$x\tan\theta\frac{dx}{d\theta} + \operatorname{cosec}^2\theta = 0,$$

for $0 < \theta < \frac{1}{2}\pi$ and $x > 0$. It is given that $x = 4$ when $\theta = \frac{1}{6}\pi$. Solve the differential equation, obtaining an expression for x in terms of θ. [6]

289. 9709_s19_qp_32 Q: 7

The variables x and y satisfy the differential equation $\frac{dy}{dx} = xe^{x+y}$. It is given that $y = 0$ when $x = 0$.

(i) Solve the differential equation, obtaining y in terms of x. [7]

(ii) Explain why x can only take values that are less than 1. [1]

290. 9709_s19_qp_33 Q: 5

The variables x and y satisfy the differential equation

$$(x+1)y\frac{dy}{dx} = y^2 + 5.$$

It is given that $y = 2$ when $x = 0$. Solve the differential equation obtaining an expression for y^2 in terms of x. [7]

291. 9709_w19_qp_31 Q: 4

The number of insects in a population t weeks after the start of observations is denoted by N. The population is decreasing at a rate proportional to $N\mathrm{e}^{-0.02t}$. The variables N and t are treated as continuous, and it is given that when $t = 0$, $N = 1000$ and $\frac{\mathrm{d}N}{\mathrm{d}t} = -10$.

(i) Show that N and t satisfy the differential equation

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -0.01\mathrm{e}^{-0.02t}N. \qquad [1]$$

(ii) Solve the differential equation and find the value of t when $N = 800$. [6]

(iii) State what happens to the value of N as t becomes large. [1]

292. 9709_w19_qp_32 Q: 6

The variables x and θ satisfy the differential equation

$$\sin\tfrac{1}{2}\theta\,\frac{dx}{d\theta} = (x+2)\cos\tfrac{1}{2}\theta$$

for $0 < \theta < \pi$. It is given that $x = 1$ when $\theta = \frac{1}{3}\pi$. Solve the differential equation and obtain an expression for x in terms of $\cos\theta$. [8]

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293. 9709_w19_qp_32 Q: 10

The line l has equation $\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$. The plane p has equation $2x + y - 3z = 5$.

(i) Find the position vector of the point of intersection of l and p. [3]

(ii) Calculate the acute angle between l and p. [3]

(iii) A second plane q is perpendicular to the plane p and contains the line l. Find the equation of q, giving your answer in the form $ax + by + cz = d$. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

294. 9709_w19_qp_33 Q: 9

The variables x and t satisfy the differential equation $5\dfrac{dx}{dt} = (20 - x)(40 - x)$. It is given that $x = 10$ when $t = 0$.

(i) Using partial fractions, solve the differential equation, obtaining an expression for x in terms of t. [9]

(ii) State what happens to the value of x when t becomes large. [1]

295. 9709_m18_qp_32 Q: 6

The variables x and θ satisfy the differential equation

$$x\cos^2\theta\frac{dx}{d\theta} = 2\tan\theta + 1,$$

for $0 \leqslant \theta < \frac{1}{2}\pi$ and $x > 0$. It is given that $x = 1$ when $\theta = \frac{1}{4}\pi$.

(i) Show that $\frac{d}{d\theta}(\tan^2\theta) = \frac{2\tan\theta}{\cos^2\theta}$. [1]

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(ii) Solve the differential equation and calculate the value of x when $\theta = \frac{1}{3}\pi$, giving your answer correct to 3 significant figures. [7]

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296. 9709_s18_qp_31 Q: 6

In a certain chemical reaction the amount, x grams, of a substance is decreasing. The differential equation relating x and t, the time in seconds since the reaction started, is

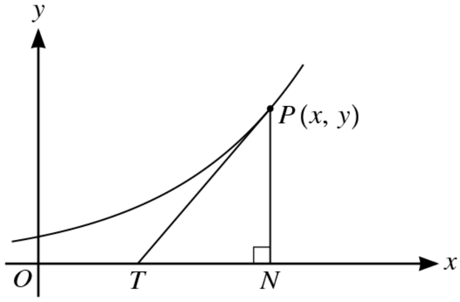
$$\frac{dx}{dt} = -kx\sqrt{t},$$

where k is a positive constant. It is given that $x = 100$ at the start of the reaction.

(i) Solve the differential equation, obtaining a relation between x, t and k. [5]

(ii) Given that $t = 25$ when $x = 80$, find the value of t when $x = 40$. [3]

297. 9709_s18_qp_32 Q: 3



In the diagram, the tangent to a curve at the point P with coordinates (x, y) meets the x-axis at T. The point N is the foot of the perpendicular from P to the x-axis. The curve is such that, for all values of x, the gradient of the curve is positive and $TN = 2$.

(i) Show that the differential equation satisfied by x and y is $\frac{dy}{dx} = \frac{1}{2}y$. [1]

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The point with coordinates $(4, 3)$ lies on the curve.

(ii) Solve the differential equation to obtain the equation of the curve, expressing y in terms of x. [5]

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298. 9709_s18_qp_33 Q: 6

(i) Express $\dfrac{1}{4-y^2}$ in partial fractions. [2]

(ii) The variables x and y satisfy the differential equation

$$x\frac{dy}{dx} = 4 - y^2,$$

and $y = 1$ when $x = 1$. Solve the differential equation, obtaining an expression for y in terms of x. [6]

299. 9709_w18_qp_31 Q: 5

The coordinates (x, y) of a general point on a curve satisfy the differential equation

$$x\frac{dy}{dx} = (2 - x^2)y.$$

The curve passes through the point $(1, 1)$. Find the equation of the curve, obtaining an expression for y in terms of x. [7]

300. 9709_w18_qp_32 Q: 6

A certain curve is such that its gradient at a general point with coordinates (x, y) is proportional to $\frac{y^2}{x}$. The curve passes through the points with coordinates (1, 1) and (e, 2). By setting up and solving a differential equation, find the equation of the curve, expressing y in terms of x. [8]

301. 9709_s17_qp_31 Q: 9

(i) Express $\dfrac{1}{x(2x+3)}$ in partial fractions. [2]

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ii) The variables x and y satisfy the differential equation

$$x(2x+3)\frac{dy}{dx} = y,$$

and it is given that $y = 1$ when $x = 1$. Solve the differential equation and calculate the value of y when $x = 9$, giving your answer correct to 3 significant figures. [7]

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302. 9709_s17_qp_32 Q: 5

In a certain chemical process a substance A reacts with and reduces a substance B. The masses of A and B at time t after the start of the process are x and y respectively. It is given that $\frac{dy}{dt} = -0.2xy$ and $x = \frac{10}{(1+t)^2}$. At the beginning of the process $y = 100$.

(i) Form a differential equation in y and t, and solve this differential equation. [6]

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(ii) Find the exact value approached by the mass of B as t becomes large. State what happens to the mass of A as t becomes large. [2]

304. 9709_w17_qp_31 Q: 6

The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = 4\cos^2 y \tan x,$$

for $0 \leqslant x < \frac{1}{2}\pi$, and $x = 0$ when $y = \frac{1}{4}\pi$. Solve this differential equation and find the value of x when $y = \frac{1}{3}\pi$. [8]

305. 9709_w17_qp_32 Q: 5

The variables x and y satisfy the differential equation

$$(x+1)\frac{dy}{dx} = y(x+2),$$

and it is given that $y = 2$ when $x = 1$. Solve the differential equation and obtain an expression for y in terms of x. [7]

306. 9709_m16_qp_32 Q: 7

The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = xe^{x+y},$$

and it is given that $y = 0$ when $x = 0$.

(i) Solve the differential equation and obtain an expression for y in terms of x. [7]

(ii) Explain briefly why x can only take values less than 1. [1]

307. 9709_s16_qp_31 Q: 4

The variables x and y satisfy the differential equation

$$x\frac{dy}{dx} = y(1 - 2x^2),$$

and it is given that $y = 2$ when $x = 1$. Solve the differential equation and obtain an expression for y in terms of x in a form not involving logarithms. [6]

308. 9709_s16_qp_32 Q: 6

The variables x and θ satisfy the differential equation

$$(3 + \cos 2\theta)\frac{dx}{d\theta} = x \sin 2\theta,$$

and it is given that $x = 3$ when $\theta = \frac{1}{4}\pi$.

(i) Solve the differential equation and obtain an expression for x in terms of θ. [7]

(ii) State the least value taken by x. [1]

309. 9709_s16_qp_33 Q: 5

The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = e^{-2y} \tan^2 x,$$

for $0 \leqslant x < \frac{1}{2}\pi$, and it is given that $y = 0$ when $x = 0$. Solve the differential equation and calculate the value of y when $x = \frac{1}{4}\pi$. [8]

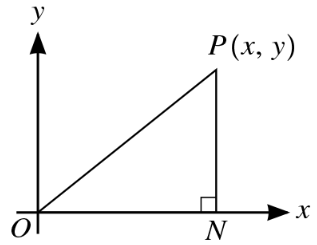
310. 9709_w16_qp_31 Q: 10

A large field of area 4 km^2 is becoming infected with a soil disease. At time t years the area infected is x km^2 and the rate of growth of the infected area is given by the differential equation $\frac{dx}{dt} = kx(4 - x)$, where k is a positive constant. It is given that when $t = 0$, $x = 0.4$ and that when $t = 2$, $x = 2$.

(i) Solve the differential equation and show that $k = \frac{1}{4} \ln 3$. [9]

(ii) Find the value of t when 90% of the area of the field is infected. [2]

311. 9709_w16_qp_33 Q: 5



The diagram shows a variable point P with coordinates (x, y) and the point N which is the foot of the perpendicular from P to the x-axis. P moves on a curve such that, for all $x \geqslant 0$, the gradient of the curve is equal in value to the area of the triangle OPN, where O is the origin.

(i) State a differential equation satisfied by x and y. [1]

The point with coordinates $(0, 2)$ lies on the curve.

(ii) Solve the differential equation to obtain the equation of the curve, expressing y in terms of x. [5]

(iii) Sketch the curve. [1]

312. 9709_s15_qp_31 Q: 7

Given that $y = 1$ when $x = 0$, solve the differential equation

$$\frac{dy}{dx} = 4x(3y^2 + 10y + 3),$$

obtaining an expression for y in terms of x. [9]

313. 9709_s15_qp_32 Q: 9

The number of organisms in a population at time t is denoted by x. Treating x as a continuous variable, the differential equation satisfied by x and t is

$$\frac{dx}{dt} = \frac{xe^{-t}}{k + e^{-t}},$$

where k is a positive constant.

(i) Given that $x = 10$ when $t = 0$, solve the differential equation, obtaining a relation between x, k and t. [6]

(ii) Given also that $x = 20$ when $t = 1$, show that $k = 1 - \frac{2}{e}$. [2]

(iii) Show that the number of organisms never reaches 48, however large t becomes. [2]

314. 9709_s15_qp_33 Q: 7

The number of micro-organisms in a population at time t is denoted by M. At any time the variation in M is assumed to satisfy the differential equation

$$\frac{dM}{dt} = k(\sqrt{M})\cos(0.02t),$$

where k is a constant and M is taken to be a continuous variable. It is given that when $t = 0$, $M = 100$.

(i) Solve the differential equation, obtaining a relation between M, k and t. [5]

(ii) Given also that $M = 196$ when $t = 50$, find the value of k. [2]

(iii) Obtain an expression for M in terms of t and find the least possible number of micro-organisms. [2]

315. 9709_w15_qp_31 Q: 8

The variables x and θ satisfy the differential equation

$$\frac{dx}{d\theta} = (x + 2)\sin^2 2\theta,$$

and it is given that $x = 0$ when $\theta = 0$. Solve the differential equation and calculate the value of x when $\theta = \frac{1}{4}\pi$, giving your answer correct to 3 significant figures. [9]

316. 9709_w15_qp_33 Q: 10

Naturalists are managing a wildlife reserve to increase the number of plants of a rare species. The number of plants at time t years is denoted by N, where N is treated as a continuous variable.

(i) It is given that the rate of increase of N with respect to t is proportional to $(N - 150)$. Write down a differential equation relating N, t and a constant of proportionality. [1]

(ii) Initially, when $t = 0$, the number of plants was 650. It was noted that, at a time when there were 900 plants, the number of plants was increasing at a rate of 60 per year. Express N in terms of t. [7]

(iii) The naturalists had a target of increasing the number of plants from 650 to 2000 within 15 years. Will this target be met? [2]